

Electrodynamic phenomena induced by a dark fluid: Analogues of pyromagnetic, piezoelectric, and striction effects

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We establish a new model of coupling between a cosmic dark fluid and electrodynamic systems, based on an analogy with effects of electric and magnetic striction, piezo-electricity and piezo-magnetism, pyro-electricity and pyro-magnetism, which appear in classical electrodynamics of continuous media. Extended master equations for electromagnetic and gravitational fields are derived using Lagrange formalism. A cosmological application of the model is considered, and it is shown that a striction-type interaction between the dark energy (the main constituent of the dark fluid) and electrodynamic system provides the universe history to include the so-called unlighted epochs, during which electromagnetic waves can not propagate and thus can not scan the universe interior.

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I. INTRODUCTION

Dark fluid composed of a dark energy and a dark matter is considered nowadays as a key constitutive element of modern cosmological models (see, e.g., [1–15]). Both the dark energy and dark matter are assumed to consist of electrically neutral particles and thus the dark fluid does not interact with an electromagnetic field *directly*. That is why, we would like, first of all, to explain the terminological context, which allows us to speak about electrodynamic phenomena induced by the dark fluid. Let us imagine a hierarchical cosmological system, in which the dark fluid (energetically dominating substrate with a modern contribution about 95%) is considered to be the guiding element, and an electrodynamic subsystem (as a part of baryonic matter with its modern contribution about 5%) to be the subordinate element. In this context the electrodynamic subsystem plays the role of a *marker*, which signalizes about the variations in the state of dark fluid, the energetic reservoir, into which this marker is immersed. We are interested to answer the question: what mechanisms might be responsible for a (possible) transmission of information about the dark fluid state to the electrodynamic system. Since electrodynamic systems form the basis for the most important channel of information about the universe structure, one could try to reconstruct features of the dark fluid evolution by tracking down specific fine details of the spectrum of observed electromagnetic waves, of their phase and group velocities.

The most known *marker-effect* of such type is the polarization rotation of electromagnetic waves traveling through the axionic dark matter [16–19]. Let us remind a few details of this phenomenon. From the physical point of view, the corresponding mechanism is connected with magneto-electric cross-effect [20, 21], which is generated in the medium by the pseudoscalar field associated with dark matter axions. From the mathematical point of view, this mechanism is described by in-

serting a special term into the Lagrangian, $\frac{1}{4}\phi F_{mn}^* F^{mn}$, which is linear in the pseudoscalar (axion) field ϕ and is proportional to the pseudo-invariant of the electromagnetic field quadratic in the Maxwell tensor [22]. The model of this axion-photon coupling was extended for the non-stationary state of the dark matter (see, e.g., [23, 24]), and for the states, for which nonminimal effects linear in the space-time curvature are significant (see, e.g., [25]).

Availability of the example of the coupling of photons with the dark matter axions encourages us to search for marker-effects related to the interaction of electrodynamic system with the dark energy, the main constituent of the dark fluid. We assume that electrodynamic systems can be influenced by the pressure of the dark energy in analogy with mechanical stresses, which are known to control the response in electric and magnetic materials in industry and technique. To be more precise, we can search for dark fluid analogies with the following classical effects. First, we mean the analogy with the classical piezo-electric effect (the appearance of an electric polarization in the medium influenced by mechanical stress and vice-versa), and with the classical piezo-magnetic effect (appearance of a magnetization under stress)(see, e.g., [26–29]). Second, we would like to consider an analogy with the inverse electrostriction effect (a combination of external pressure and electric field generates the electric polarization in the medium), and an analogy with the inverse magnetostriction effect (a combination of external pressure and magnetic field generates the magnetization in the medium), as well as, an analogy with the magneto-electric cross-effect displayed by the external stress. Based on results of classical electrodynamics of continuous media, we can expect that piezo-effects will be visualized, when the dark fluid is anisotropic (e.g., in the early universe). The striction effects due to their symmetries are expected to be available in the isotropic universe also. In addition to piezo- and striction- effects induced by the dark fluid pressure we can expect the appearance of marker-signals similar to pyro-electric and pyro-magnetic responses of the medium, in which the temperature changes with time [26–28] (pyro-effect also is hidden, when the dark fluid is isotropic). One of the important characteristic of the

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dark fluid is its macroscopic velocity four-vector and covariant derivative of this four-vector. When we focus on the influence of the dark fluid non-uniform motion on the properties of electrodynamic system, we, in fact, search for analogs of dynamo-optical phenomena [28]; we hope to consider these phenomena in detail in the next paper. In principle, we could consider an analogy with the so-called thermo-electric and thermo-magnetic effects, induced by heat-fluxes in the medium, but this sector of physical modeling is out of scope of this paper. Also, in this paper we do not consider magneto-electric cross-effects induced by the combination of the dark energy pressure and of the axionic dark matter. Effects of this type are worthy of special consideration.

We have to emphasize that mathematical theory of pyro-, piezo- and striction- effects is developed in detail for classical electrodynamics of continuous media, and below we consider a general relativistic extension of that theory for the case of dark fluid action on the electrodynamic system. In this sense, we take the mathematical scheme of the description of such interactions, which is well-tested, has clear interpretation and is based on the Lagrange formalism, and then we construct its general relativistic analog, using this scheme in the context of dark fluid electrodynamics.

This paper is organized as follows. In Section II we remind the terminology and introduce the Lagrangian and master equations for the model of electromagnetically inactive dark fluid. Section III contains detailed description of the extended model: in Section III.A we extend the Lagrangian by the terms, which describe interactions of the pyro-, piezo- and striction- types between dark energy and electrodynamic system; in Section III.B we derive extended electrodynamic equations and discuss the structure of tensor coefficients describing pyro- (III.B.1), piezo- (III.B.2) and striction- (III.B.3) coefficients associated with the coupling to dark energy; in Section III.C we obtain the extended gravity field equations in general form. In Section III.D we write the equation for a axion field attributed to the dark matter. In Section IV we reduce the derived master equations to the case, when the medium is spatially isotropic: Section IV.A contains details of reduced electrodynamic equations; in Section IV.B we collect details of modified gravity field equations; in Section IV.C we consider an example of extended model with hidden magnetic and/or electric anisotropy. In Section V we consider a cosmological application of the established model to the problem of description of the so-called unlighted epochs in the universe history, and their relations to the striction-type interactions of electrodynamic systems with the cosmic dark energy. In Section VI we summarize the results. Appendix includes working formulas for the extended variation procedure.

II. ELECTROMAGNETICALLY INACTIVE DARK FLUID

In order to remind the standard elements of the theory and to introduce new details, let us start with the model, in the framework of which the electromagnetic field interacts with the standard matter only, and the

dark fluid is coupled with the electrodynamic system by the gravitational field only.

A. The Lagrangian

Let us remind that the standard Einstein-Maxwell model is described by the action functional

$$S_0 = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + L_{(\text{DF})} + \frac{1}{4} C_{(0)}^{ikmn} F_{ik} F_{mn} + L_{(\text{m})} \right], \quad (1)$$

which is quadratic in the Maxwell tensor F_{ik} . Here g is the determinant of the metric tensor g_{ik} , the term R is the Ricci scalar, $\kappa = \frac{8\pi G}{c^4}$ is the Einstein constant, $L_{(\text{DF})}$ is the Lagrangian of the dark fluid, $L_{(\text{m})}$ is the Lagrangian of a standard matter. The tensor $C_{(0)}^{ikmn}$ describes standard linear electromagnetic response of the medium formed by the standard matter; in vacuum the corresponding term takes the form $\frac{1}{4} F^{mn} F_{mn}$. As usual, we assume that $L_{(\text{m})}$ does not include the Maxwell tensor F_{mn} , nevertheless, it can depend on the potential four-vector A_i , if the medium is conductive.

B. Standard electrodynamic equations

The Maxwell tensor is represented in terms of a four-vector potential A_i as

$$F_{ik} = \nabla_i A_k - \nabla_k A_i, \quad (2)$$

and thus satisfies the condition

$$\nabla_k F^{*ik} = 0, \quad (3)$$

where $F^{*ik} \equiv \frac{1}{2} \epsilon^{ikpq} F_{pq}$ is the tensor dual to F_{pq} , the term $\epsilon^{ikpq} \equiv \frac{1}{\sqrt{-g}} E^{ikpq}$ is the Levi-Civita tensor, E^{ikpq} is the absolutely skew-symmetric Levi-Civita symbol with $E^{0123} = 1$. The variation of the action functional (1) with respect to the four-vector potential A_i gives the following electrodynamic equations

$$\nabla_k H^{ik} = -\frac{4\pi}{c} I^i, \quad H^{ik} = C_{(0)}^{ikmn} F_{mn}. \quad (4)$$

Here H^{ik} is the excitation tensor [21], and the four-vector I^i defined as

$$I^i = \frac{1}{4\pi} \frac{\delta L_{(\text{m})}}{\delta A_i}, \quad \nabla_i I^i = 0, \quad (5)$$

describes the electric current. In this paper we consider the medium to be non-conducting, i.e., $I^i = 0$.

C. Gravity field equations

Variation of the action functional (1) with respect to metric gives the equations of the gravitational field, which can be written in the following form

$$R_{ik} - \frac{1}{2} R g_{ik} = \kappa \left[T_{ik}^{(0)} + T_{ik}^{(\text{DE})} + T_{ik}^{(\text{DM})} + T_{ik}^{(\text{m})} \right]. \quad (6)$$

Here $T_{ik}^{(0)}$ is the effective symmetric traceless stress-energy tensor of electromagnetic field in a continuous medium

$$T_{ik}^{(0)} \equiv \left[\frac{1}{4} g_{ik} F_{mn} - \frac{1}{2} (g_{im} F_{kn} + g_{km} F_{in}) \right] C_{(0)}^{mnpq} F_{pq}, \quad (7)$$

(see, e.g., [30, 31] for details). The Lagrangian of the electromagnetically inactive dark fluid is presented as a sum $L_{(\text{DF})} \rightarrow L_{(\text{DE})} + L_{(\text{DM})}$, and the corresponding stress-energy tensors enter the right-hand side of equations (6) also as the sum. The stress-energy tensor of the dark energy is defined as

$$T_{ik}^{(\text{DE})} = -\frac{2}{\sqrt{-g}} \frac{\delta [\sqrt{-g} L_{(\text{DE})}]}{\delta g^{ik}}. \quad (8)$$

Let us define the unit four-vector U^i of the macroscopic velocity as an eigen-vector of the stress-energy tensor of the dark energy, i.e., let us assume that $T_{ls}^{(\text{DE})} U^s = W U_l$, $U^i U_i = 1$ (the so-called Landau-Lifshitz definition). Then this tensor can be algebraically represented in the following form

$$T_{ik}^{(\text{DE})} \equiv W U_i U_k + \mathcal{P}_{ik}, \quad (9)$$

where the eigen-value W is interpreted as the energy density of the dark energy, and \mathcal{P}_{ik} is its pressure tensor. These quantities can be written as follows:

$$W = U^l T_{ls}^{(\text{DE})} U^s, \quad \mathcal{P}_{ik} = \Delta_i^l T_{ls}^{(\text{DE})} \Delta_k^s, \quad (10)$$

where $\Delta_i^l \equiv \delta_i^l - U^l U_i$ is the projector. The tensors $T_{ik}^{(\text{DM})}$ and $T_{ik}^{(\text{m})}$, which describe the contributions of the dark matter and standard matter, respectively, can be obtained by the formulas similar to (8), however, their algebraic decompositions are more sophisticated, since the macroscopic velocity four-vector U^i is already fixed as an eigen-vector of the dark energy stress-energy tensor. For instance, $T_{ik}^{(\text{m})}$ is of the form

$$T_{ik}^{(\text{m})} \equiv W^{(\text{m})} U_i U_k + U_i I_k^{(\text{m})} + U_k I_i^{(\text{m})} + P_{ik}^{(\text{m})}, \quad (11)$$

and includes the heat-flux four-vector $I_k^{(\text{m})} \equiv \Delta_k^l T_{ls}^{(\text{m})} U^s$ of the matter in addition to the matter energy-density $W^{(\text{m})}$ and the matter pressure tensor $P_{ik}^{(\text{m})}$. When we describe the dark matter, one should change the symbol (m) by (DM) in the formula (11).

III. ELECTROMAGNETICALLY ACTIVE DARK FLUID

A. Extended Lagrangian

Let us extend the action functional to include the terms describing the interaction between the dark fluid and electrodynamic system. We assume the extension to have the following form

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa} + L_{(\text{DE})} + \frac{1}{4} C_{(0)}^{ikmn} F_{ik} F_{mn} + L_{(\text{m})} + \right.$$

$$\begin{aligned} & + \frac{1}{2} \Psi_0^2 [-\nabla_k \phi \nabla^k \phi + \mathcal{V}(\phi^2)] + \frac{1}{4} \phi F_{mn}^* F^{mn} + \\ & + \frac{1}{2} \left(\pi^{ik} + \frac{1}{2} \lambda^{ikmn} F_{mn} \right) F_{ik} DW + \\ & \left. + \frac{1}{2} \left(\mathcal{D}^{ikpq} + \frac{1}{2} Q^{ikmnpq} F_{mn} \right) F_{ik} \mathcal{P}_{pq} \right\}, \quad (12) \end{aligned}$$

which again is up to second order in the Maxwell tensor F_{ik} , but in addition to the second order terms also the terms linear in F_{mn} appeared. Here and below we use the symbol D for the convective derivative $D \equiv U^i \nabla_i$. We specified the Lagrangian of the dark matter $L_{(\text{DM})}$ as the one for a pseudoscalar (axion) field ϕ ; in this context the quantity $\frac{1}{\Psi_0}$ is a coupling constant of the axion-photon interaction, and $\mathcal{V}(\phi^2)$ is the potential of the pseudoscalar field. In this terms, the cross-invariant $\frac{1}{4} \phi F_{mn}^* F^{mn}$ describes the coupling between the electromagnetic and pseudoscalar fields, i.e., the interaction between axionic dark matter and electromagnetic field [22]. In other words, we deal here with the example of electromagnetically active dark fluid, and this type of activity is connected with the dark matter part of the dark fluid.

Now we consider the dark fluid activity related to the interaction of the electromagnetic field with dark energy constituent of the dark fluid. Based on the analogy with electrodynamics of continuous media we can consider the first term linear in the Maxwell tensor, $\frac{1}{2} \pi^{ik} F_{ik} DW$, as describing the analog of the pyro effects (pyro-electric and/or pyro-magnetic). Of course, in classical electrodynamics of continuous media one deals with the convective derivative of the temperature DT , when one speaks about pyro - effects, nevertheless, assuming that $DW = \frac{dW}{dT} DT$ we keep this terminology for the dark fluid also and indicate the tensor π^{ik} as the tensor of pyro-coefficients. The second term linear in the Maxwell tensor, $\frac{1}{2} \mathcal{D}^{ikpq} F_{ik} \mathcal{P}_{pq}$, includes the pressure tensor of the dark energy \mathcal{P}_{pq} and thus describes analogs of piezo-effects (piezo-electric and/or piezo-magnetic). The corresponding piezo-coefficients are encoded in the tensor \mathcal{D}^{ikpq} . The term quadratic in F_{mn} and linear in DW describes the part of the linear electromagnetic response, which depends on the rate of evolution of the energy density of the dark fluid. The last term in (12) is quadratic in the Maxwell tensor and linear in the pressure tensor, thus describing the response associated with electro- and magneto-striction, induced by the dark energy; the tensor Q^{ikmnpq} introduces coefficients of electro- and magneto-striction.

B. Extended electrodynamic equations

The variation of the action functional (12) with respect to the four-vector potential A_i gives the following electrodynamic equations

$$\nabla_k H^{ik} = -\frac{4\pi}{c} I^i, \quad H^{ik} = \mathcal{H}^{ik} + \phi F^{*ik} + C^{ikmn} F_{mn}. \quad (13)$$

Here H^{ik} is the extended excitation tensor. The term

$$\mathcal{H}^{ik} \equiv \pi^{ik} DW + \mathcal{D}^{ikpq} \mathcal{P}_{pq} \quad (14)$$

does not contain F_{ik} and thus it can be indicated as the spontaneous polarization-magnetization tensor. The term ϕF^{*ik} is typical for the axion electrodynamics (see, e.g., [22]); the four-divergence of this term can be expressed as $F^{*ik} \nabla_k \phi$ due to (3). Finally, the term

$$\mathcal{C}^{ikmn} \equiv C_{(0)}^{ikmn} + \lambda^{ikmn} DW + Q^{ikmnpq} \mathcal{P}_{pq} \quad (15)$$

describes the total linear response of the electrodynamic system including pyro-type and striction-type effects induced by the dark energy.

1. Pyro- coefficients associated with the coupling to dark energy

The skew-symmetric tensor π^{ik} describing the pyro- effects can be represented as

$$\pi^{ik} = \pi^i U^k - \pi^k U^i - \epsilon^{ik}_{mn} \mu^m U^n, \quad (16)$$

thus visualizing the pyro-electric π^i and pyro-magnetic μ^m coefficients, which are orthogonal to the velocity four-vector ($\pi^i U_i = 0 = \mu^m U_m$). In general case the dark energy can be characterized by three pyro-electric and three pyro-magnetic coefficients. When we consider the dark energy as a spatially isotropic medium, all six pyro coefficients vanish. For the dark energy with an axial symmetry (e.g., in rotationally symmetric Bianchi-I model) there are two non-vanishing piezo-constants: one piezo-electric and one piezo-magnetic (see, e.g., [26, 27] for details).

2. Piezo-coefficients attributed to the coupling to dark energy

The tensor \mathcal{D}^{ikpq} possesses the following symmetry of indices

$$\mathcal{D}^{ikpq} = -\mathcal{D}^{kipq} = \mathcal{D}^{ikqp}. \quad (17)$$

Since the symmetric pressure tensor \mathcal{P}_{pq} is considered to be orthogonal to the velocity four-vector U^i , one can conclude that

$$\mathcal{D}^{ikpq} U_p = 0 = \mathcal{D}^{ikpq} U_q. \quad (18)$$

This means that there are $6 \times 6 = 36$ independent coupling constants in the tensor of piezo-coefficients. This tensor can be decomposed with respect to irreducible parts as follows:

$$\mathcal{D}^{ikpq} = d^{i(pq)} U^k - d^{k(pq)} U^i - \epsilon^{ik}_{ls} U^s h^{l(pq)}. \quad (19)$$

Here the piezo-electric coefficients $d^{i(pq)}$ and piezo-magnetic coefficients $h^{l(pq)}$ are defined by

$$d^{i(pq)} \equiv \mathcal{D}^{ikpq} U_k, \quad h^{l(pq)} \equiv \frac{1}{2} \epsilon^{ls}_{ik} \mathcal{D}^{ikpq} U_s, \quad (20)$$

they are symmetric with respect to the indices p, q , and are pure space-like, i.e., they satisfy the equalities

$$d^{i(pq)} U_i = 0 = d^{i(pq)} U_p, \quad h^{l(pq)} U_l = 0 = h^{l(pq)} U_p. \quad (21)$$

In other words, in general case, the dark energy influence can be characterized by 18 piezo-electric coefficients $d^{i(pq)}$ and/or by 18 piezo-magnetic coefficients $h^{l(pq)}$. When the dark energy is spatially isotropic, all these coefficients are equal to zero. For the dark energy with axial symmetry there are four non-vanishing piezo-electric and four non-vanishing piezo-magnetic coefficients (see, e.g., [26, 27] for details).

3. Permittivity tensors associated with the coupling to dark energy

Using the medium velocity four-vector U^i one can decompose \mathcal{C}^{ikmn} uniquely as

$$\begin{aligned} \mathcal{C}^{ikmn} = & \left(\epsilon^{i[m} U^{n]} U^k - \epsilon^{k[m} U^{n]} U^i \right) - \\ & - \frac{1}{2} \eta^{ikl} (\mu^{-1})_{ls} \eta^{mns} + \eta^{ikl} U^{[m} \nu_l^{n]} + \eta^{lmn} U^{[i} \nu_l^{k]}. \end{aligned} \quad (22)$$

Here ϵ^{im} is the tensor of total dielectric permeability, $(\mu^{-1})_{pq}$ is the tensor of total magnetic impermeability, and ν_p^m is the total tensor of magneto-electric cross-effect induced by dark energy. Keeping in mind the decomposition (15) one can divide these quantities into three parts

$$\epsilon^{im} = 2\mathcal{C}^{ikmn} U_k U_n = \epsilon_{(0)}^{im} + \sigma^{im} DW + \alpha^{im(pq)} \mathcal{P}_{pq},$$

$$(\mu^{-1})^{ab} = -\frac{1}{2} \eta^a_{ik} \mathcal{C}^{ikmn} \eta_{mn}^b =$$

$$= (\mu^{-1})_{(0)}^{ab} + \rho^{ab} DW + \beta^{ab(pq)} \mathcal{P}_{pq},$$

$$\nu^{am} = \eta^a_{ik} \mathcal{C}^{ikmn} U_n = \nu_{(0)}^{am} + \omega^{am} DW + \gamma^{am(pq)} \mathcal{P}_{pq}. \quad (23)$$

To complete the description of the tensor of cross-effects, we can write the sum of ν^{am} related to the dark energy contribution (23) and $\nu_{(\text{DM})}^{am} = \phi \Delta^{am}$ related to the contribution of the axionic dark matter. In the formulas written above we introduced the corresponding spatial tensors as follows:

$$\epsilon_{(0)}^{im} = 2\mathcal{C}_{(0)}^{ikmn} U_k U_n, \quad \sigma^{im} = 2\lambda^{ikmn} U_k U_n,$$

$$\alpha^{im(pq)} = 2Q^{ikmnpq} U_k U_n,$$

$$(\mu^{-1})_{(0)}^{ls} = -\frac{1}{2} \eta^l_{ik} \mathcal{C}_{(0)}^{ikmn} \eta_{mn}^s,$$

$$\rho^{ls} = -\frac{1}{2} \eta^l_{ik} \lambda^{ikmn} \eta_{mn}^s,$$

$$\beta^{ls(pq)} = -\frac{1}{2}\eta_{ik}^l Q^{ikmnpq} \eta_{mn}^s, \quad \nu_{(0)}^{am} = \eta_{ik}^a C_{(0)}^{ikmn} U_n,$$

$$\omega^{am} = \eta_{ik}^a \lambda^{ikmn} U_n, \quad \gamma^{lm(pq)} = \eta_{ik}^l Q^{ikmnpq} U_n. \quad (24)$$

As usual, the tensors η_{mnl} and η^{ikl} are the skew-symmetric and orthogonal to U^i ; they are defined as

$$\eta_{mnl} \equiv \epsilon_{mnl} U^s, \quad \eta^{ikl} \equiv \epsilon^{ikls} U_s. \quad (25)$$

These tensors are connected by the useful identity

$$-\eta^{ikp} \eta_{mnp} = \delta_{mns}^{ikl} U_l U^s = \Delta_m^i \Delta_n^k - \Delta_n^i \Delta_m^k. \quad (26)$$

Upon contraction, equation (26) yields another useful identity

$$\frac{1}{2} \eta^{ikl} \eta_{klm} = -\delta_{ms}^{il} U_l U^s = -\Delta_m^i. \quad (27)$$

The quantities δ_{mns}^{ikl} and δ_{ms}^{il} are the generalized Kronecker deltas. Clearly, the two-indices tensors $\varepsilon_{(0)}^{im}$, $(\mu^{-1})_{ab}^{(0)}$, σ^{im} , ρ^{ab} are symmetric and orthogonal to U_k ; each of them possesses six independent components. The spatial (pseudo)tensors $\nu_{(0)}^{am}$ and ω^{am} are non-symmetric and thus each of them contains nine independent components.

The spatial tensor $\alpha^{im(pq)}$ possesses the symmetry

$$\alpha^{im(pq)} = \alpha^{mi(pq)} = \alpha^{im(qp)}, \quad (28)$$

and, generally, it has $6 \times 6 = 36$ independent components. When the medium is spatially isotropic there are only two scalars representing this tensor. The symmetry of the tensor $\beta^{ls(pq)}$ is similar:

$$\beta^{ls(pq)} = \beta^{sl(pq)} = \beta^{ls(qp)}, \quad (29)$$

and it also has 36 independent components in general case, and only two parameters in the spatially isotropic case. Finally, the tensor $\gamma^{lm(pq)}$ with the symmetry

$$\gamma^{lm(pq)} = \gamma^{lm(qp)}, \quad (30)$$

is characterized by $9 \times 6 = 54$ independent components in general case, and vanishes in the spatially isotropic medium. Using (24), we see that the tensor Q^{abmnpq} is generally characterized by 126 independent components.

C. Gravity field equations

The extended gravity field equations

$$\frac{1}{\kappa} \left[R_{ik} - \frac{1}{2} R g_{ik} \right] =$$

$$= T_{ik}^{(0)} + T_{ik}^{(\text{DE})} + T_{ik}^{(\text{DM})} + T_{ik}^{(\text{m})} + T_{ik}^{(\text{W})} + T_{ik}^{(\text{P})} + T_{ik}^{(\text{S})} \quad (31)$$

contain the stress-energy tensor of the electromagnetic field in the material medium $T_{ik}^{(0)}$ defined by (7), the stress-energy tensor of the dark energy $T_{ik}^{(\text{DE})}$ presented by (9), the stress-energy tensor of the standard matter

$T_{ik}^{(\text{m})}$ decomposed as (11), the stress-energy tensor of the pseudoscalar (axion) field $T_{ik}^{(\text{DM})}$ given by

$$T_{ik}^{(\text{DM})} = \nabla_i \phi \nabla_k \phi - \frac{1}{2} g_{ik} \nabla^n \phi \nabla_n \phi + \frac{1}{2} g_{ik} \mathcal{V}(\phi^2), \quad (32)$$

and three new interaction terms. The term $T_{ik}^{(\text{W})}$ connected with the pyro-type interactions is of the form

$$\begin{aligned} T_{ik}^{(\text{W})} = & DW \left[\frac{1}{2} g_{ik} F_{mn} \left(\pi^{mn} + \frac{1}{2} F_{ls} \lambda^{mnl} \right) - \right. \\ & \left. - F_{mn} \left(\frac{\delta}{\delta g^{ik}} \pi^{mn} + \frac{1}{2} F_{ls} \frac{\delta}{\delta g^{ik}} \lambda^{mnl} \right) \right] + \\ & + \left[W \left(\frac{1}{2} g_{ik} + U_i U_k \right) - 2 \mathcal{B}_{ikls} U^l U^s \right] \times \\ & \times \nabla_j \left[U^j F_{mn} \left(\pi^{mn} + \frac{1}{2} F_{ls} \lambda^{mnl} \right) \right] - \\ & - \frac{1}{2} F_{mn} \left(\pi^{mn} + \frac{1}{2} F_{ls} \lambda^{mnl} \right) U_{(i} \nabla_{k)} W. \end{aligned} \quad (33)$$

The term $T_{ik}^{(\text{P})}$ relates to the piezo-type contribution to the total stress-energy tensor of the system; it has the form

$$\begin{aligned} T_{ik}^{(\text{P})} = & \frac{1}{2} g_{ik} \mathcal{D}^{mnpq} F_{mn} \mathcal{P}_{pq} + 2 \mathcal{D}^{mnpq} F_{mn} \mathcal{B}_{ikls} \Delta_p^l \Delta_q^s - \\ & - F_{mn} \mathcal{P}_{ls} \frac{\delta}{\delta g^{ik}} (\mathcal{D}^{mnpq} \Delta_p^l \Delta_q^s). \end{aligned} \quad (34)$$

The last new term describes the contribution of the striction - type interactions; it can be written as

$$\begin{aligned} T_{ik}^{(\text{S})} = & \frac{1}{4} g_{ik} Q^{abmnpq} F_{ab} F_{mn} \mathcal{P}_{pq} - \\ & - \frac{1}{2} F_{ab} F_{mn} \mathcal{P}_{ls} \frac{\delta}{\delta g^{ik}} (Q^{abmnpq} \Delta_p^l \Delta_q^s) + \\ & + Q^{abmnpq} F_{ab} F_{mn} \mathcal{B}_{ikls} \Delta_p^l \Delta_q^s. \end{aligned} \quad (35)$$

The tensor \mathcal{B}_{ikls} in the formulas (33), (34) and (35) is defined as follows

$$\mathcal{B}_{ikls} \equiv \frac{1}{\sqrt{-g}} \frac{\delta^2}{\delta g^{ik} \delta g^{ls}} [\sqrt{-g} L_{(\text{DE})}]. \quad (36)$$

This four-indices tensor has the following symmetry:

$$\mathcal{B}_{ikls} = \mathcal{B}_{kils} = \mathcal{B}_{iksl} = \mathcal{B}_{lsik}. \quad (37)$$

It can be decomposed phenomenologically using the similar algebraic procedure as for the decomposition of the stress-energy tensor of the dark energy (8):

$$\mathcal{B}_{ikls} = \mathcal{G} U_i U_k U_l U_s + \left(\mathcal{G}_{ls}^{(1)} U_i U_k + \mathcal{G}_{ik}^{(1)} U_l U_s \right) +$$

$$\begin{aligned}
& + (\mathcal{G}_i U_k U_l U_s + \mathcal{G}_k U_i U_l U_s + \mathcal{G}_l U_i U_k U_s + \mathcal{G}_s U_i U_k U_l) + \\
& + \left(\mathcal{G}_{ks}^{(2)} U_i U_l + \mathcal{G}_{kl}^{(2)} U_i U_s + \mathcal{G}_{is}^{(2)} U_k U_l + \mathcal{G}_{il}^{(2)} U_k U_s \right) + \\
& + (\mathcal{G}_{ikl} U_s + \mathcal{G}_{iks} U_l + \mathcal{G}_{kls} U_i + \mathcal{G}_{ils} U_k) + \mathcal{G}_{ikls}, \quad (38)
\end{aligned}$$

where

$$\mathcal{G} \equiv U^a U^b \mathcal{B}_{abcd} U^c U^d, \quad \mathcal{G}_s \equiv U^a U^b \mathcal{B}_{abcd} U^c \Delta_s^d,$$

$$\mathcal{G}_{ls}^{(1)} \equiv U^a U^b \mathcal{B}_{abcd} \Delta_l^c \Delta_s^d,$$

$$\mathcal{G}_{ks}^{(2)} \equiv U^a U^c \mathcal{B}_{abcd} \Delta_k^b \Delta_s^d, \quad \mathcal{G}_{kls} \equiv U^a \mathcal{B}_{abcd} \Delta_k^b \Delta_l^c \Delta_s^d,$$

$$\mathcal{G}_{ikls} \equiv \mathcal{B}_{abcd} \Delta_i^a \Delta_k^b \Delta_l^c \Delta_s^d. \quad (39)$$

The projected four-indices tensor appeared in (34), (35)

$$\mathcal{B}_{ikls} \Delta_p^l \Delta_q^s = \mathcal{G}_{pq}^{(1)} U_i U_k + \mathcal{G}_{kpq} U_i + \mathcal{G}_{ipq} U_k + \mathcal{G}_{ikpq} \quad (40)$$

contains only four terms. The two-indices tensor contributed to (33)

$$\mathcal{B}_{ikls} U^l U^s = \mathcal{G} U_i U_k + \mathcal{G}_i U_k + \mathcal{G}_k U_i + \mathcal{G}_{ik}^{(1)}, \quad (41)$$

also includes only four terms. We will calculate directly the tensors \mathcal{G} , \mathcal{G}_k , $\mathcal{G}_{ls}^{(1)}$, $\mathcal{G}_{ks}^{(2)}$, \mathcal{G}_{ikl} and \mathcal{G}_{ikls} below for the model with spatial isotropy.

D. Equations for the pseudoscalar (axion) field

Since we represented the dark matter constituent of the dark fluid by a pseudoscalar (axion) field, we can easily derive the evolutionary equation for the dark matter by variation of the action functional (12) with respect to the pseudoscalar ϕ ; this procedure yields

$$[\nabla^k \nabla_k + \mathcal{V}'(\phi^2)] \phi = -\frac{1}{4\Psi_0^2} F_{mn}^* F^{mn}. \quad (42)$$

In [23–25] we studied more sophisticated equations describing the interaction between electromagnetic field and axionic dark matter; nevertheless, here we restrict ourselves by this simplest model of the axion-photon coupling.

E. Short summary

We derived the set of coupled master equations for the extended model: first, electrodynamic equations (13)–(15); second, gravity field equations (31)–(41); third, equations for the axion field (42). Of course, the derivation of these equations is only the first step in our program. In the next paper we plan to apply these equations for the description of anisotropic models of early universe (in particular, to the Bianchi-I model with global magnetic field) and to consider an anisotropic dark energy (in analogy with, e.g., [32]). For these applications pyro- and piezo- effects seem to be important. Below we consider only one application of the established model, namely, the application to the isotropic homogeneous model of the Friedmann type. We hope it will be a good illustration that the established model is worthy of attention.

IV. MASTER EQUATIONS FOR A SPATIALLY ISOTROPIC MEDIUM

During the late-time universe evolution the dark fluid is considered as a spatially isotropic substratum. The dark matter is modelled as a cold substratum with vanishing pressure. The pressure tensor of such dark fluid is proportional to the projector, $\mathcal{P}_{ik} = -P\Delta_{ik}$, and the scalar quantity P describes the pressure of the dark energy. Concerning the symmetry of tensor coefficients in the context of isotropic and homogeneous cosmological model, we have to assume that all pyro- and piezo- coefficients are vanishing. In addition, we have to assume that all the non-vanishing tensor coefficients can be constructed using three basic elements: first, pure geometrical quantities (metric g_{ik} , Levi-Civita tensor ϵ^{ikmn} , Kronecker deltas δ_k^i , δ_{mn}^{ik} , etc.); second, the dynamic quantity U^i (macroscopic velocity of the dark energy), the projector Δ_{ik} ; third, phenomenologically introduced coupling constants (in front of corresponding terms). We can calculate directly the variation of all such quantities with respect to the metric, thus completing the model reconstruction. Let us consider in detail the model with spatial isotropy.

A. Reduction of the electrodynamic equations

In the spatially isotropic medium we have to use the tensor of linear response in the following form:

$$C_{(0)}^{ikmn} = \frac{1}{2\mu_{(0)}} [g^{ikmn} + (\varepsilon_{(0)}\mu_{(0)} - 1)(g^{ikmn} - \Delta^{ikmn})]. \quad (43)$$

Here the scalars $\varepsilon_{(0)}$ and $\mu_{(0)}$ are dielectric and magnetic permittivities of the medium, respectively, in the case when the dark fluid influence on this medium is negligible. Similarly, the tensor λ^{ikmn} is of the form

$$\lambda^{ikmn} = \frac{1}{2} [\lambda_2 g^{ikmn} + (\lambda_1 - \lambda_2)(g^{ikmn} - \Delta^{ikmn})], \quad (44)$$

where two phenomenological constants λ_1 and λ_2 introduce contributions of the pyro-type interactions (proportional to DW) into the total linear response tensor. Because of spatial isotropy the tensor of pyro-coefficients π^{ik} and the tensor of piezo-coefficients \mathcal{D}^{ikmn} vanish.

The last non-trivial element of the extended theory is the tensor of striction-type activity Q^{ikmnpq} ; in order to construct it we will write, first of all, the space-like tensors $\alpha^{im(pq)}$ and $\beta^{im(pq)}$ using their symmetry:

$$\alpha^{im(pq)} = \alpha_{(1)} \Delta^{im} \Delta^{pq} + \alpha_{(2)} (\Delta^{ip} \Delta^{mq} + \Delta^{iq} \Delta^{mp}), \quad (45)$$

$$\beta^{im(pq)} = \beta_{(1)} \Delta^{im} \Delta^{pq} + \beta_{(2)} (\Delta^{ip} \Delta^{mq} + \Delta^{iq} \Delta^{mp}). \quad (46)$$

As for the (pseudo)tensor $\gamma^{im(pq)}$, keeping in mind the analogy with classical electrodynamics of spatially isotropic continuous media [26, 27], we assume that this (pseudo)tensor of cross-effects is equal to zero, i.e., $\gamma^{im(pq)} = 0$. In other words, when the medium is spatially isotropic, we deal with four coupling parameters

$\alpha_{(1)}$, $\alpha_{(2)}$, $\beta_{(1)}$, $\beta_{(2)}$, which characterize electro- and magneto-striction. The corresponding reconstruction of the tensor Q^{ikmnpq} yields

$$\begin{aligned} Q^{ikmnpq} = & \frac{1}{2} [\alpha_{(1)} \Delta^{pq} (g^{ikmn} - \Delta^{ikmn}) + \\ & + \alpha_{(2)} U_l U_s (g^{iklp} g^{mnsq} + g^{iklq} g^{mns p}) + \\ & + \beta_{(1)} \Delta^{pq} \Delta^{ikmn} - \beta_{(2)} (\eta^{ikp} \eta^{mnq} + \eta^{ikq} \eta^{mnp})] . \end{aligned} \quad (47)$$

Clearly, only the four-indices tensor $Q^{ikmnpq} \Delta_{pq}$ enters the electrodynamic equations, when the pressure tensor of the dark energy is spatially isotropic; it has now the form

$$Q^{ikmn} \equiv Q^{ikmnpq} \Delta_{pq} = \frac{1}{2} \alpha g^{ikmn} + \frac{1}{2} (\beta - \alpha) \Delta^{ikmn} , \quad (48)$$

i.e., only two effective coupling constants

$$\alpha = 3\alpha_{(1)} + 2\alpha_{(2)} , \quad \beta = 3\beta_{(1)} + 2\beta_{(2)} , \quad (49)$$

appeared in this tensor instead of four parameters $\alpha_{(1)}$, $\alpha_{(2)}$, $\beta_{(1)}$ and $\beta_{(2)}$. Total permittivity tensors of the spatially isotropic medium influenced by the dark energy are now the following:

$$\varepsilon^{im} = \Delta^{im} \varepsilon , \quad \varepsilon = \varepsilon_{(0)} + \lambda_1 DW - \alpha P , \quad (50)$$

$$(\mu^{-1})_{ab} = \frac{1}{\mu} \Delta_{ab} , \quad \frac{1}{\mu} = \frac{1}{\mu_{(0)}} + \lambda_2 DW - \beta P , \quad (51)$$

$$\nu^{am} = 0 . \quad (52)$$

This means that in the spatially isotropic case one can define the scalar refraction index of the medium, n , accounting for the influence of the dark energy, yielding

$$n^2 \equiv \varepsilon \mu = \frac{n_{(0)}^2 + \mu_{(0)} (\lambda_1 DW - \alpha P)}{1 + \mu_{(0)} (\lambda_2 DW - \beta P)} , \quad (53)$$

where $n_{(0)}^2 \equiv \varepsilon_{(0)} \mu_{(0)}$ is the square of the refraction index of the medium which does not feel the dark energy influence. The corresponding phase velocity of the electromagnetic waves in the striction-active medium is

$$V_{(ph)} \equiv \frac{c}{n} = c \sqrt{\frac{1 + \mu_{(0)} (\lambda_1 DW - \beta P)}{n_{(0)}^2 + \mu_{(0)} (\lambda_2 DW - \alpha P)}} . \quad (54)$$

The group velocity of the electromagnetic waves is defined as

$$V_{(gr)} \equiv c \frac{2n}{(n^2 + 1)} , \quad (55)$$

(see, e.g., [33] for details), and can be easily displayed using (53). Let us mention that the influence of the dark energy provides the spatially isotropic electrodynamic system to possess non-stationary properties, when the universe expands. To be more precise, the refraction index, the phase and group velocities become functions of the cosmological time: $n(t)$, $V_{(ph)}(t)$, $V_{(gr)}(t)$, due to the coupling to the non-stationary dark energy with time dependent energy density $W(t)$ and pressure $P(t)$.

B. Reduction of the gravity field equations

In order to reduce the formulas (33)-(36) for the spatially isotropic case we have to make the following preliminary steps. First, we put the tensors π^{ik} , \mathcal{D}^{ikpq} equal to zero, since now the spontaneous polarization-magnetization is inadmissible because of the model symmetry. The second step is to calculate directly the variation derivatives $\frac{\delta}{\delta g^{ik}} \lambda^{mnl}$, $\frac{\delta}{\delta g^{ik}} (Q^{abmnpq} \Delta_p^l \Delta_q^s)$ using the reduced formulas (44), (47) and the auxiliary formulas, which we presented in Appendix. The third step is to derive the formulas for the variation derivatives $\frac{\delta}{\delta g^{ik}} W$, $\frac{\delta}{\delta g^{ik}} DW$ and $\frac{\delta}{\delta g^{ik}} P$. Let us consider in detail the third step.

Our ansatz is that the *spatially isotropic* dark energy can be modelled by a real scalar field Ψ with the Lagrangian

$$L_{(DE)} = -\frac{1}{2} g^{mn} \partial_m \Psi \partial_n \Psi + \frac{1}{2} V(\Psi^2) , \quad (56)$$

and the corresponding stress-energy tensor $T_{ik}^{(DE)}$

$$T_{ik}^{(DE)} = \partial_i \Psi \partial_k \Psi - \frac{1}{2} g_{ik} g^{mn} \partial_m \Psi \partial_n \Psi + \frac{1}{2} g_{ik} V(\Psi^2) . \quad (57)$$

This idea correlates with the attempt to describe the dark matter in terms of pseudoscalar field ϕ (see (12) and (32)). The velocity four-vector U^i is defined as an eigen-vector of the tensor $T_{ik}^{(DE)}$, and we obtain readily

$$T_{ik}^{(DE)} U^k = W U_i =$$

$$= \partial_i \Psi D \Psi - \frac{1}{2} U_i g^{mn} \partial_m \Psi \partial_n \Psi + \frac{1}{2} U_i V(\Psi^2) , \quad (58)$$

$$W \equiv U^i T_{ik}^{(DE)} U^k = (D \Psi)^2 - \frac{1}{2} g^{mn} \partial_m \Psi \partial_n \Psi + \frac{1}{2} V(\Psi^2) . \quad (59)$$

Thus, $\partial_n \Psi = U_n D \Psi$ and we obtain the well-known relationships (see, e.g., [34]):

$$W = \frac{1}{2} [(D \Psi)^2 + V(\Psi^2)] ,$$

$$P \equiv -\frac{1}{3} \Delta^{im} T_{ik}^{(DE)} \Delta_m^k = \frac{1}{2} [(D \Psi)^2 - V(\Psi^2)] , \quad (60)$$

which give (see auxiliary formulas in Appendix)

$$\begin{aligned} \frac{\delta}{\delta g^{ik}} W &= \frac{\delta}{\delta g^{ik}} P = \frac{1}{2} \partial_i \Psi \partial_k \Psi = \\ &= \frac{1}{2} U_i U_k (D \Psi)^2 = \frac{1}{2} U_i U_k (W + P) . \end{aligned} \quad (61)$$

Similarly, direct calculation of the tensor \mathcal{B}_{ikls} yields

$$\begin{aligned} \mathcal{B}_{ikls} &= \frac{1}{4} (2W - P) U_i U_k U_l U_s + \\ &+ \frac{1}{4} W (U_i U_k \Delta_{ls} + U_l U_s \Delta_{ik}) - P U_{(l} \Delta_{s)(k} U_{i)} - \end{aligned}$$

$$-\frac{1}{4}P(\Delta_{ls}\Delta_{ik}+\Delta_{li}\Delta_{ks}+\Delta_{lk}\Delta_{is}). \quad (62)$$

Thus, for this illustrative example we obtain

$$\begin{aligned} \mathcal{G} &= \frac{1}{4}(2W-P), \quad \mathcal{G}_s = 0, \\ \mathcal{G}_{ls}^{(1)} &= \frac{1}{4}W\Delta_{ls}, \quad \mathcal{G}_{ks}^{(2)} = -\frac{1}{4}P\Delta_{ls}, \quad \mathcal{G}_{kls} = 0, \\ \mathcal{G}_{ikls} &= -\frac{1}{4}P(\Delta_{ls}\Delta_{ik}+\Delta_{li}\Delta_{ks}+\Delta_{lk}\Delta_{is}). \end{aligned} \quad (63)$$

These formulas give the hint to write the working formula

$$\begin{aligned} &\frac{\delta}{\delta g^{ik}}\mathcal{P}_{pq} = \\ &= \frac{1}{2}U_i U_k [\mathcal{P}_{pq} - \Delta_{pq}W] - \frac{1}{2}[g_{p(i}\mathcal{P}_{k)q} + g_{q(i}\mathcal{P}_{k)p}], \end{aligned} \quad (64)$$

and then to summarize the total stress-energy tensor of the electromagnetic field interacting with the dark energy as follows:

$$\begin{aligned} T_{ik}^{(\text{EM})} &= T_{ik}^{(0)} + T_{ik}^{(\text{W})} + T_{ik}^{(\text{S})} = \\ &= \left[\frac{1}{4}g_{ik}F_{mn}F_{mn} - \frac{1}{2}(g_{im}F_{kn} + g_{km}F_{in}) \right] \mathcal{C}^{mnab}F_{ab} + \\ &+ \frac{1}{8}U_i U_k F_{ab}F_{mn} \{ (W+P)Q^{abmn} + \\ &+ \lambda^{abmn} [(W+P)\nabla_j U^j - DW] \} + \\ &+ \frac{1}{8}U_i U_k (W+P)D(\lambda^{abmn}F_{ab}F_{mn}). \end{aligned} \quad (65)$$

Here we used the notations

$$\begin{aligned} \mathcal{C}^{abmn} &= C_{(0)}^{abmn} + \lambda^{abmn} DW - Q^{abmn} P = \\ &= \frac{1}{2\mu} [g^{abmn} + (\varepsilon\mu - 1)(g^{abmn} - \Delta^{abmn})], \end{aligned} \quad (66)$$

where the quantities μ and ε are already defined by (50) and (51).

C. Model with hidden anisotropy

The spatial isotropy of the space-time happens to be violated, when one considers the model with global magnetic and/or electric fields. For the magnetic field in vacuum one should study the Bianchi-I cosmological model instead of the Friedmann one, since the tensor $\Delta_i^p T_{pq}^{(\text{EM})} \Delta_k^q$ in the right-hand side of the Einstein equations is not spatially isotropic in this case (i.e., $\Delta^{1p} T_{pq}^{(\text{EM})} \Delta_1^q = \Delta^{2p} T_{pq}^{(\text{EM})} \Delta_2^q \neq \Delta^{3p} T_{pq}^{(\text{EM})} \Delta_3^q$, when x^3

is the anisotropy axis). When an electrodynamic system interacts with dark energy, the situation changes essentially: one can find such states of the system, for which the magnetic field is non-vanishing but the spatial isotropy is inherited. For instance, when

$$\frac{1}{\mu_{(0)}} + \lambda_2 DW - \beta P = 0, \quad (67)$$

we obtain immediately that

$$\begin{aligned} &\Delta_i^p T_{pq}^{(\text{EM})} \Delta_k^q = \\ &= (\varepsilon_{(0)} + \lambda_1 DW - \alpha P) \left[\frac{1}{2} \Delta_{ik} E_m E^m - E_i E_k \right], \end{aligned} \quad (68)$$

where $E^i = F^{ik} U_k$ is the electric field four-vector; for pure magnetic field $E^i = 0$, thus $\Delta_i^p T_{pq}^{(\text{EM})} \Delta_k^q = 0$. This model with *hidden* magnetic anisotropy is exotic, since the effective refraction index is now equal to infinity, and the phase and group velocities of the electromagnetic waves are equal to zero for such dark medium. Similarly, when $\varepsilon_{(0)} + \lambda_1 DW - \alpha P = 0$, but $\frac{1}{\mu_{(0)}} \neq \beta P - \lambda_2 DW$ we deal with the so-called hidden electric anisotropy. Finally, when $\varepsilon_{(0)} = \alpha P - \lambda_1 DW$ and $\frac{1}{\mu_{(0)}} = \beta P - \lambda_2 DW$ simultaneously, the stress-energy tensor $\Delta_i^p T_{pq}^{(\text{EM})} \Delta_k^q$ is equal to zero identically, and the electromagnetic source of the gravity field disappears from the Einstein equations. Similar results related to a hidden magnetic anisotropy were obtained early in the frameworks of the nonminimal Einstein-Maxwell theory [35] and extended Einstein-Maxwell theory [30, 31].

V. COSMOLOGICAL APPLICATIONS: UNLIGHTED EPOCHS

We obtained extended master equations for the coupled electromagnetic and gravitational fields, which take into account interactions of pyro-, piezo- and striction- types. In the nearest future we intend to analyze applications of these master equations to the early universe with global magnetic field, and to the problem of late-time universe accelerated expansion. In this work we consider only one illustration of the extended model. To be more precise, in this Section we assume that the space-time is of the Friedmann-Lemaître-Robertson-Walker type with the metric

$$ds^2 = c^2 dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (69)$$

and this space-time is a fixed background for a local (test) electrodynamic system. In other words, here we neglect by the backreaction of the electromagnetic field on the gravity field, but consider the striction-type influence of the spatially isotropic cosmic dark energy on the electrodynamic system. We are interested in the analysis of the so-called unlighted epochs in the universe history, analogs of which were described in [33] in the framework of nonminimal field theory. We use the term "unlighted epochs" for intervals of the universe evolution, for which the square of the effective

refraction index $n^2(t)$ (see (53)) takes negative values, $n^2(t) < 0$. During these periods of time the refraction index is a pure imaginary quantity, and the phase and group velocities of the electromagnetic waves (see (54)) and (55) are not defined. Clearly, the function $n^2(t)$ can change sign at the moments $t_{(s)}$ of the cosmological time when the numerator or the denominator in (53) vanish. When the numerator vanishes, one has $n(t_{(s)})=0$, $V_{(ph)}(t_{(s)})=\infty$, and $V_{(gr)}(t_{(s)})=0$. When the denominator vanishes, one has $n(t_{(s)})=\infty$, $V_{(ph)}(t_{(s)})=0$, and $V_{(gr)}(t_{(s)})=0$. In both cases the unlighted epochs appear and disappear when the group velocity of the electromagnetic waves vanishes, i.e., at these points the energy transfer stops. We indicate the times $t_{(s)}$ as the unlighted epochs boundary points (see [33] for details). Below we consider three very illustrative examples of the evolution of the cosmic dark energy and of the corresponding behavior of the pressure function $P(t)$ for the case, when the striction coefficients only are non-vanishing.

A. De Sitter-type models

The simplest model of the dark energy is the de Sitter one; for this model the dark energy pressure is constant $P=-\Lambda$. Clearly, the refraction index for this model is also constant

$$n^2 = \frac{n_{(0)}^2 + \mu_{(0)}\alpha\Lambda}{1 + \mu_{(0)}\beta\Lambda}, \quad (70)$$

i.e., unlighted epochs are not available. When

$$\beta - \alpha = \frac{n_{(0)}^2 - 1}{\mu_{(0)}\Lambda}, \quad (71)$$

we obtain that the universe expansion is characterized by the condition $n^2=1$, so that $V_{(gr)}=V_{(ph)}=c$.

B. Anti-Gaussian solution

In [36, 37] the exact solution of the Archimedean-type model is obtained, which was indicated as Anti-Gaussian solution, since for this solution the scale factor $a(t)$ is of the form

$$a(t) = a(t^*) \exp \left[\frac{8\pi G}{3\nu} (t - t^*)^2 \right]. \quad (72)$$

Here we repeated the notations from [36]

$$\log \frac{a(t^*)}{a(t_0)} \equiv -\frac{\nu}{4} [\rho(t_0) + E_{(0)}],$$

$$t^* = t_0 - \nu \sqrt{\frac{3}{32\pi G} [\rho(t_0) + E_{(0)}]}. \quad (73)$$

The parameter ν is a coupling constant of the Archimedean-type interaction between dark energy and dark matter, the parameters $a(t_0)$, $\rho(t_0)$ and $E_{(0)}$ are the initial data for the scale factor, energy-density of

the dark energy and energy-density of the dark matter, respectively. According to that model the dark energy pressure and energy-density (here and below we use the symbols $\Pi(t)$ and $\rho(t)$, respectively, for these quantities in analogy with [36, 37]) are described by the formulas

$$\Pi(t) = \Pi(t_0) - \frac{4}{\nu} \log \left[\frac{a(t)}{a(t_0)} \right],$$

$$\rho(t) = \rho(t_0) + \frac{4}{\nu} \log \left[\frac{a(t)}{a(t_0)} \right]. \quad (74)$$

The formula for the pressure can be rewritten in the form

$$\Pi(t) = \Pi(t^*) - \frac{32\pi G}{3\nu^2} (t - t^*)^2,$$

$$\Pi(t^*) = \Pi(t_0) + \rho(t_0) + E_{(0)}. \quad (75)$$

We assume that the late-time universe evolution is characterized by the model with $n^2(t) \rightarrow 1$; this assumption provides that at present the light propagates with phase and group velocities equal to the speed of light in vacuum. According to (53) and (75) this requirement at $t \rightarrow \infty$ leads to the equality $\beta=\alpha$. Also, we put $\mu_{(0)}=1$ for simplicity and assume that $n_{(0)}^2=\varepsilon_{(0)} > 1$. Then we use the replacement

$$z^2 \equiv \frac{32\pi G}{3\nu^2} (t - t^*)^2 \quad (76)$$

and transform (53) into

$$n^2 = \frac{z^2 - Z_2}{z^2 - Z_1}, \quad (77)$$

where

$$Z_2 \equiv \Pi(t^*) - \frac{n_{(0)}^2}{\alpha}, \quad Z_1 \equiv \Pi(t^*) - \frac{1}{\alpha},$$

$$\alpha (Z_1 - Z_2) = [n_{(0)}^2 - 1] > 0. \quad (78)$$

Now we are ready to describe unlighted epochs.

1. Models without unlighted epochs

The refraction index $n(t)$ is equal to one identically, when $n_{(0)}^2=1$ and thus $Z_1=Z_2$. Also, the quantity $n^2(t)$ is positive for *arbitrary* time, when Z_1 and Z_2 are negative, i.e.,

$$\Pi(t^*) < \frac{n_{(0)}^2}{\alpha}, \quad \Pi(t^*) < \frac{1}{\alpha}. \quad (79)$$

In both cases the unlighted epochs can not appear.

2. Unlighted epochs of the first type

Let the parameter Z_1 be negative, and Z_2 be positive. Clearly, it is possible, when

$$\frac{n_{(0)}^2}{\alpha} < \Pi(t^*) < \frac{1}{\alpha} \Rightarrow \alpha < 0, \quad \Pi(t^*) < 0. \quad (80)$$

Thus, the refraction index is imaginary, when $|z| < \sqrt{Z_2}$, or in more details,

$$|t - t^*| < \Delta_2, \quad \Delta_2 \equiv \sqrt{\frac{3\nu^2}{32\pi G} \left[\Pi(t^*) - \frac{n_{(0)}^2}{\alpha} \right]}. \quad (81)$$

At the boundary points of this time interval, $t_{(\pm)} = t^* \pm \Delta_2$, the refraction index and the group velocity vanish, $n(t_{(\pm)}) = 0$, $V_{(gr)}(t_{(\pm)}) = 0$, and the phase velocity becomes infinite $V_{(ph)}(t_{(\pm)}) = \infty$. The duration of this unlighted epoch is equal to $2\Delta_2$ (see Panel A of Fig.1).

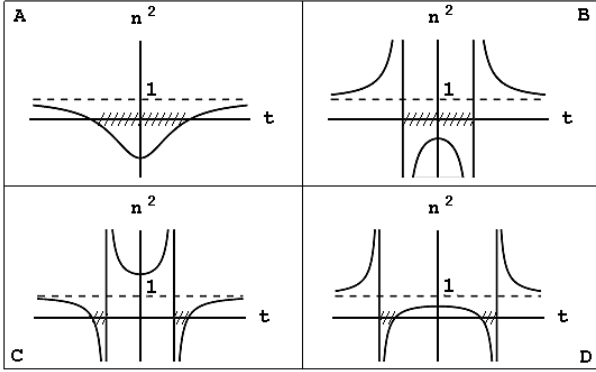


FIG. 1: Sketches of basic graphs illustrating unlighted epochs of four types. Unlighted epochs relate to the intervals of cosmological time t for which the function $n^2(t)$, the squared effective refraction index, is negative. Panel A relates to the case, when $Z_1 < 0$, $Z_2 > 0$, and thus the denominator of the function $n^2(t)$ is positive (see (76)-(78)); this panel illustrates the simply connected unlighted epoch of the first type with zeroth values of the function $n^2(t)$ at the boundary points. Panel B illustrates the simply connected unlighted epoch of the second type, which corresponds to the case $Z_1 > 0$, $Z_2 < 0$, so that both boundary values of the function $n^2(t)$ are infinite. The graphs of the unlighted epochs of the third type (Panel C) and of the fourth type (Panel D) are doubly-connected; they correspond to the cases $Z_2 > Z_1 > 0$ and $Z_1 > Z_2 > 0$, respectively. When $n^2=0$ or $n^2=\infty$, the group velocity of electromagnetic wave (see (55)) takes zero value, i.e., the energy transfer stops. For all the cases one has that $n^2(t \rightarrow \pm\infty) \rightarrow 1$ (see (dashed) horizontal asymptotes.)

3. Unlighted epochs of the second type

Now, let the parameter Z_2 be negative, and Z_1 be positive. Clearly, it is possible, when

$$\frac{1}{\alpha} < \Pi(t^*) < \frac{n_{(0)}^2}{\alpha} \Rightarrow \alpha > 0, \quad \Pi(t^*) > 0. \quad (82)$$

Thus, the refraction index is imaginary, when $|z| < \sqrt{Z_1}$, or in more details,

$$|t - t^*| < \Delta_1, \quad \Delta_1 \equiv \sqrt{\frac{3\nu^2}{32\pi G} \left[\Pi(t^*) - \frac{1}{\alpha} \right]}. \quad (83)$$

At the boundary points of this time interval, $t_{(\pm)} = t^* \pm \Delta_1$, the refraction index is infinite, $n(t_{(\pm)}) = \infty$, thus the group and phase velocities vanish, $V_{(gr)}(t_{(\pm)}) = 0$, $V_{(ph)}(t_{(\pm)}) = 0$. The duration of this unlighted epoch is equal to $2\Delta_1$ (see Panel B of Fig.1).

4. Unlighted epochs of the third type

Now we consider the case, when both parameters Z_1 and Z_2 are positive. For positive α this gives the conditions

$$\Pi(t^*) > \frac{n_{(0)}^2}{\alpha}, \quad (84)$$

so that $n^2(t) < 0$, when $\sqrt{Z_2} < |z| < \sqrt{Z_1}$, or equivalently,

$$\Delta_2 < |t - t^*| < \Delta_1. \quad (85)$$

This unlighted epoch is divided into two separated sub-epochs, the duration of both sub-epochs is $\Delta_1 - \Delta_2$ (see Panel C of Fig.1). Similarly, when $\alpha < 0$ and $\Pi(t^*) > \frac{1}{\alpha}$, we could find two unlighted sub-epochs at $\Delta_1 < |t - t^*| < \Delta_2$ (see Panel D of Fig.1).

C. Super-exponential solution

In [36] the exact solution of the Archimedean-type model is obtained, which was indicated as super-exponential, since for this solution the scale factor is of the form

$$\frac{a(t)}{a(t_0)} = \exp \left\{ \sqrt{\frac{2\rho(t_0)}{9\rho_0}} \sinh \left[\sqrt{12\pi G\rho_0} (t - t_0) \right] \right\}, \quad (86)$$

where the parameter ρ_0 is the so-called bag constant. The corresponding dark energy pressure is

$$\begin{aligned} \Pi(t) = & \Pi(t_0) + 3[\rho(t_0) + \Pi(t_0) - \rho_0] \log \left[\frac{a(t)}{a(t_0)} \right] - \\ & - \frac{9}{2} \rho_0 \log^2 \left[\frac{a(t)}{a(t_0)} \right]. \end{aligned} \quad (87)$$

Surprisingly, this model can be reduced to the one considered in the previous section, if we use the following definition for z :

$$\begin{aligned} z(t) \equiv & \sqrt{\rho(t_0)} \sinh \left[\sqrt{12\pi G\rho_0} (t - t_0) \right] - \\ & - \frac{[\rho(t_0) + \Pi(t_0) - \rho_0]}{\sqrt{2\rho_0}}. \end{aligned} \quad (88)$$

In order to complete the analogy, we find the parameter t^* from the equation $z(t^*)=0$ (see (88)) and obtain from (87) and (86) that

$$\Pi(t^*) \equiv \Pi(t_0) + \frac{[\rho(t_0) + \Pi(t_0) - \rho_0]^2}{2\rho_0}, \quad (89)$$

with Z_1 and Z_2 inherited from (78). Thus, we deal again with the analysis of the formula (77) and obtain the similar results, if we make the replacement

$$\sqrt{\frac{32\pi G}{3\nu^2}}(t - t^*) \Rightarrow \left\{ \sqrt{\rho(t_0)} \sinh \left[\sqrt{12\pi G \rho_0}(t - t_0) \right] - \frac{[\rho(t_0) + \Pi(t_0) - \rho_0]}{\sqrt{2\rho_0}} \right\}. \quad (90)$$

Again, the model admits the existence of unlighted epochs of four types sketched on Panels 1-4 of Fig.1.

D. Remarks on stability of the dark energy scalar potential under quantum fluctuations, and constraints on the striction model

We established pure classical model of striction-type interactions between dark energy and electrodynamic system. The paper context does not allow us to consider quantum aspects of this model, however, we hope to return to this problem in the next work. Here we would like to discuss briefly only three remarks, which could be important for physical understanding of the model consequences.

1. How the corrections to the scalar field potential can influence the master equations of the striction-type model?

The crucial point of establishing the total set of master equations of the striction-type model is the finding of the tensor \mathcal{B}_{ikls} (see (36) and (38)). Generally, it can not be presented in an explicit form by means of the energy density W and of the dark energy pressure P , and its reconstruction requires sophisticated phenomenological decomposition (38). However, when we treat the dark energy as a spatially homogeneous isotropic medium using an analogy with some scalar field Ψ , we obtain \mathcal{B}_{ikls} by direct variation procedure (see (62)), thus providing the model to be self-closed. Moreover, we find that, when we restrict our-selves by the striction-type interactions only, i.e., $\lambda^{ikmn}=0$, the electrodynamic equations include the scalar P only, and the gravity field equations include P and the combination $W+P$ only. It follows from (60) that the sum $W+P=(D\Psi)^2$ does not feel the shape of the potential $V(\Psi^2)$; as for the dark energy pressure P , it, clearly, depends on $V(\Psi^2)$. This fact confirms that modeling of striction-type extensions of master equations for the electromagnetic and gravitational fields by means of a scalar dark energy is sensitive to the choice of the scalar field potential.

2. One-loop corrections to the scalar field potential in the framework of anti-Gaussian model

Based on the method of one-loop corrections to potentials of complex or real scalar fields attributed to the dark energy, dark matter or dark fluid, the authors of the works [38, 39] have shown that these scalar field potentials are usually stable under quantum fluctuations, however, a coupling to fermions is very restricted. When we consider an electrodynamic system influenced by a scalar dark energy, we have to take into account these results keeping in mind two aspects. First, the scalar field potential $V(\Psi^2)$ attributed to the dark energy (see (56)) can be modified due to one-loop corrections along the line discussed in [38, 39]; since this potential enters the formula for the pressure of the dark energy (see (60)), these corrections can re-define the coupling constants of the striction-type interaction discussed above. Second, the electrodynamic system inevitably contains electrically charged fermions; when the fermion mass m_f depends on the scalar field, $m_f(\Psi)$, we can use directly the method and estimations discussed in [38, 39]. As for (possible) dependence of the photon mass on the scalar field Ψ , this problem is worthy a special attention and will be discussed in a separate paper.

Let us apply the method used in [38, 39] to the anti-Gaussian model discussed in Subsection V.B. According to (74) and (60)), we readily obtain that

$$\dot{\Psi}^2(t) = \rho(t) + \Pi(t) = \rho(t_0) + \Pi(t_0) \equiv \dot{\Psi}^2(t_0), \quad (91)$$

thus providing the scalar field Ψ to be linear function of the cosmological time t :

$$\Psi(t) = \Psi(t_0) + \dot{\Psi}(t_0) t. \quad (92)$$

Then, using (59)) and (60)) we see that the potential $V(\Psi^2)=V^*(\Psi)$ is a quadratic function of the scalar field:

$$V^*(\Psi) = A + 2B\Psi + C\Psi^2, \quad (93)$$

where

$$A = \rho(t_0) - \Pi(t_0) - 2B\Psi(t_0) - C\Psi^2(t_0),$$

$$B = \frac{16}{\nu\dot{\Psi}(t_0)} \sqrt{\frac{\pi G}{6}} [\rho(t_0) + E_{(0)}] - C\Psi(t_0),$$

$$C = \frac{64\pi G}{3\nu^2\dot{\Psi}^2(t_0)}. \quad (94)$$

For the function (93) the second derivative of the potential is $V''(\Psi)=2C$, thus the formula (2) from [38] gives us the one-loop modification of the scalar potential (93)

$$V_{1-\text{loop}}(\Psi) = V^*(\Psi) + \frac{4G\Lambda_s^2}{3\nu^2\dot{\Psi}^2(t_0)} - \frac{\Lambda_f^2}{8\pi^2} [m_f(\Psi)]^2, \quad (95)$$

where Λ_s and Λ_f are the ultra-violet cutoffs of the scalar and fermion fluctuations, respectively; $m_f(\Psi)$ is Ψ -dependent fermion mass, which appears in the Lagrangian of matter $L_{(m)}$ (see (1)), when the matter

is considered to consist of fermions (see (1) in [38]). Clearly, being constant, the second term in the right-hand side of (95) can be absorbed by the constant A appeared in the potential (93); the authors of the works [38, 39] indicate such a case as describing the potential stable under scalar fluctuations. As for estimations of the potential stability related to fermion fluctuations, the third term in (95) is exactly the same as the one in (2) of the paper [38]; this means that estimations discussed in Section III of [38] are also valid. Let us repeat, that estimations related to the scalar field coupling to photons and gravitons based on one-loop corrections require a special consideration and, unfortunately, are out of frame of this paper.

3. *How the corrections to the scalar field potential could be visualized in the cosmic striction-type phenomena?*

Let us consider the consequences of the scalar field potential modification, given by the second term in the right-hand side of (95), for the unlighted epochs formation. In fact, this constant one-loop correction to the quadratic potential leads to the following formal re-definition of the parameter $\Pi(t^*)$ in (75):

$$\Pi(t^*) \rightarrow \tilde{\Pi}(t^*) = \Pi(t^*) - \frac{2G\Lambda_s^2}{3\pi\nu^2\dot{\Psi}^2(t_0)}. \quad (96)$$

Then we have to replace the parameter $\Pi(t^*)$ with this new parameter $\tilde{\Pi}(t^*)$ in all inequalities, which predetermine the structure of the unlighted epochs (see Sections V.B1, V.B2, V.B3, V.B4). Let us illustrate the consequences of such pressure re-definition by the examples, for which $\alpha > 0$ and $\Pi(t^*)$ is positive (see, e.g., (79), (82), (84)). Since the second term in (96) is negative, we can imagine that $\tilde{\Pi}(t^*)$ becomes negative due to such scalar potential correction. This means that the inequalities (79) (with the replacement $\Pi(t^*) \rightarrow \tilde{\Pi}(t^*)$) remain valid, so that this veto for the unlighted epoch appearance holds out. The inequalities (82) and (84) are not valid now, therefore, the corresponding unlighted epochs of the second and third types become forbidden because of scalar fluctuations. In other words, the answer on the question about absence or presence of the unlighted epochs seems to be very sensitive to the choice of the scalar field potential and to its one-loop corrections.

VI. DISCUSSION AND CONCLUSIONS

1. A new model of coupling between a cosmic dark fluid and electrodynamic systems is established, i.e., the extended Lagrangian is introduced and the extended master equations are derived for electromagnetic and gravitational fields. What is the novelty of this model from the mathematical point of view? First, we introduced four cross-terms into the Lagrangian, which contain the Maxwell tensor up to the second order, on the one hand, and contain the pressure tensor of the dark energy and the convective derivative of its energy-density scalar, on the other hand. Thus, a modified

Lagrangian is not of pure field-type, since these cross-terms are the products of pure field-type elements (F_{ik}) and of quantities defined algebraically (see (9), (10) for the definitions of U^i , W , \mathcal{P}_{pq}). Second, we described a modified procedure of variation with respect to the metric, which happened to be necessary for these new cross-terms defined algebraically. This modified procedure is based on the rule of variation of the macroscopic velocity four-vector (99) taken from the works [30, 31]; on the rule of algebraic decomposition of the second variation of the dark energy Lagrangian (36)-(39), and on the ansatz about the structure of this decomposition in the case of spatially isotropic medium (see (61)-(64)).

2. What is the physical motivation of the Lagrangian extension, which we made? Since we follow the mathematical scheme, which is well-known in the relativistic electrodynamics of continuous media, we indicate the new cross-terms in the extended Lagrangian (12) using the similar terminology: as (dark energy inspired) analogs of terms describing electric and magnetic striction, piezo-electricity and piezo-magnetism, pyro-electricity and pyro-magnetism. This analogy allowed us to interpret new coupling constants as (dark energy induced) pyro-, piezo- and striction coefficients, respectively.

3. First cosmological application of the model shows that a striction-type interaction between the dark energy and test electrodynamic system provides the phase and group velocities of electromagnetic waves to become sophisticated functions of cosmological time. In the asymptotic regime, at $t \rightarrow \infty$, these functions tend to the speed of light in vacuum, i.e., $V_{(\text{ph})} \rightarrow c$ and $V_{(\text{gr})} \rightarrow c$. However, during the universe evolution the so-called unlighted epochs can appear, for which the effective refraction index of the cosmic medium is an imaginary quantity. At the boundary points of these unlighted epochs the group velocity of the electromagnetic waves takes zero value, so the electromagnetic energy transfer stops.

4. The last unlighted epoch (if such epochs have ever existed in the real Universe) should finish before the so-called recombination era, as far as, cosmic microwave background composed of relic photons traveling freely is observable. In our terms this means that the characteristic time $t_{(\text{lastUE})} = t^* + \Delta_2$ is less than $t_{(\text{rec})} \simeq 10^{13}$ sec. Using (73), (81), (75) at $t_0 = t_{(\text{rec})}$ and the condition $t_{(\text{lastUE})} < t_{(\text{rec})}$, we obtain a cosmological constraint $\Pi(t_{(\text{rec})}) < \frac{n^2(t_{(\text{rec})})}{\alpha}$ linking the phenomenological parameter of a striction-type coupling α , the value of the refraction index $n(t_{(\text{rec})})$ at the end of the recombination era, and the value of the dark energy pressure $\Pi(t_{(\text{rec})})$ at that moment. It is only one example of constraints appearing in this model; we intend to discuss other constraints in a special note.

5. The criteria of existence of the unlighted epochs are very sensitive to the choice of the scalar field potential, which one uses for modeling of the dark energy. We have illustrated this sentence on the example of the anti-Gaussian model. In particular, taking into account one-loop corrections to the dark energy scalar field potential, one can show, that scalar fluctuations are able

to avoid the unlighted epoch formation.

6. We expect that the established model can provide new interesting results in application to the anisotropic Bianchi-I cosmological model, since in this period of the universe evolution pyro-magnetic and piezo-magnetic effects can appear in addition to the magneto- striction effect admissible both on the anisotropic and isotropic stages of the universe expansion.

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Appendix

In order to calculate directly the variation derivatives of basic quantities, we have to fix the following auxiliary formulas. We start with the well-known formulas for the variation of the determinant of the metric and metric itself:

$$\frac{1}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{ik}} = -\frac{1}{2} g_{ik} ,$$

$$\frac{\delta g_{pq}}{\delta g^{ik}} = -\frac{1}{2} [g_{p(i} g_{k)q} + g_{q(i} g_{k)p}] , \quad (97)$$

and for the four-indices tensor $g^{ikmn} \equiv g^{im} g^{kn} - g^{in} g^{km}$

$$\frac{\delta g^{abmn}}{\delta g^{ik}} = \delta_{(i}^{[a} g_{k)}^{b]mn} + g^{ab[m} \delta_{(i}^{n]} . \quad (98)$$

The formulas for the variation of the velocity four-vector

$$\frac{\delta U^l}{\delta g^{ik}} = \frac{1}{4} (\delta_i^l U_k + \delta_k^l U_i) = \frac{1}{2} \delta_{(i}^l U_{k)} ,$$

$$\frac{\delta U_a}{\delta g^{ik}} = -\frac{1}{4} (g_{ia} U_k + g_{ka} U_i) = -\frac{1}{2} g_{a(i} U_{k)} , \quad (99)$$

stand to keep the normalization condition $g^{ik} U_i U_k = 1$ (see, e.g., [30, 31]). Using (97) and (99) it is easy to write the formulas for the variation of the projectors

$$\frac{\delta \Delta^{pq}}{\delta g^{ik}} = \delta_{(i}^{(p} \Delta_{k)}^{q)} ,$$

$$\frac{\delta \Delta_{pq}}{\delta g^{ik}} = -\frac{1}{2} [g_{p(i} \Delta_{k)q} + g_{q(i} \Delta_{k)p}] , \quad (100)$$

$$\frac{\delta \Delta_q^p}{\delta g^{ik}} = \frac{1}{2} [\delta_{(i}^p \Delta_{k)q} - \Delta_{(i}^p g_{k)q}] . \quad (101)$$

For the variation of the four-indices projector $\Delta^{ikmn} \equiv \Delta^{im} \Delta^{kn} - \Delta^{in} \Delta^{km}$ we use the convenient formula

$$\frac{\delta \Delta^{abmn}}{\delta g^{ik}} = \delta_{(i}^{[a} \Delta_{k)}^{b]mn} + \Delta^{ab[m} \delta_{(i}^{n]} . \quad (102)$$

Finally, the variation of the Levi-Civita tensor ϵ^{abpq} and of the associated tensor $\eta^{abp} \equiv \epsilon^{abpq} U_q$ yield

$$\frac{\delta \epsilon^{abpq}}{\delta g^{ik}} = \frac{1}{2} \epsilon^{abpq} g_{ik} ,$$

$$\frac{\delta \eta^{abp}}{\delta g^{ik}} = \frac{1}{2} [\eta^{abp} g_{ik} - \epsilon^{abp}_{(i} U_{k)}] . \quad (103)$$

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